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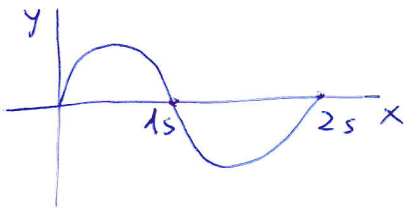
Muelle oscilando con MAS



$T = 2\,s$

$v_{max} = 0'1\,\pi\ m/s$

$e = ?$

$v = ?$

$a = ?$

$e = A \cdot \operatorname{sen}(\omega t + t_0)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi\ rad/s$

$v_{max} = A \cdot \omega \Rightarrow A = \frac{v_{max}}{\omega} = \frac{0'1\pi}{\pi} = 0'1\,m$

Sustituimos las condiciones iniciales $t=1s$ para calcular t_0. Para ello tomo la ecuación de la velocidad

$v = A \cdot \omega \cdot \cos(\omega t + t_0)$

$0'1\pi = 0'1 \cdot \pi \cdot \cos(\pi \cdot 1 + t_0)$

$\cos(\pi + t_0) = 1 \Rightarrow \cos\theta = 1$

$\theta = \arccos 1 \Rightarrow \theta = 0 \Rightarrow \pi + t_0 = 0 \Rightarrow \boxed{t_0 = -\pi\ rad}$

$x = A \cdot \operatorname{sen}(\omega t + t_0)$

$v = A \cdot \omega \cdot \cos(\omega t + t_0)$

$a = -A \cdot \omega^2 \cdot \operatorname{sen}(\omega t + t_0)$

$\boxed{x(t) = 0'1 \cdot \operatorname{sen}(\pi t - \pi)\,m} \quad \boxed{v(t) = 0'1\pi \cdot \cos(\pi t - \pi)\,m/s} \quad \boxed{a(t) = -0'1\pi^2 \cdot \operatorname{sen}(\pi t - \pi)\,\frac{m}{s^2}}$

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$v(0) = 0'6\,m/s$

$v_{max} = 1'2\,m/s$

$a_{max} = 14'4\,m/s^2$

MAS

$T = ?$

$A = ?$

$x(t)$, $v(t)$, $a(t)$

$v_{max} = A \cdot \omega$

$a_{max} = A \cdot \omega^2$

dividiendo $\Rightarrow \frac{a_{max}}{v_{max}} = \frac{A \cdot \omega^2}{A \cdot \omega}$

$\omega = \frac{a_{max}}{v_{max}} = \frac{14'4}{1'2} = 12\ rad/s$

$\omega = \frac{2\pi}{T} \Rightarrow \boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{12} = 0'52\,s}$

$v_{max} = A \cdot \omega \Rightarrow \boxed{A = \frac{v_{max}}{\omega} = \frac{1'2}{12} = 0'1\,m}$

La fase inicial la calculamos a partir de los datos iniciales

$v(t) = A \cdot \omega \cdot \cos(\omega t + t_0) \Rightarrow v(0) = 0'1 \cdot 12 \cdot \cos(12 \cdot 0 + t_0)$

$\cos t_0 = 0'5 \Rightarrow \arccos 0'5 = \frac{\pi}{3}\ rad \Rightarrow t_0 = \pi/3\ rad$

$\boxed{x(t) = 0'1 \cdot \operatorname{sen}(12t + \frac{\pi}{3})\,m} \quad \boxed{v(t) = 1'2 \cdot \cos(12t + \frac{\pi}{3})\,\frac{m}{s}} \quad \boxed{a(t) = -14'4 \cdot \operatorname{sen}(12t + \frac{\pi}{3})\ m/s^2}$

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$A = 20\,cm = 0'2\,m$; $x_0 = 5\,cm = 0'05\,m$

$a = -9\pi^2 \cdot x$
$a = -\omega^2 \cdot x$
$\Rightarrow \omega^2 = 9\pi^2 \Rightarrow \omega = \sqrt{9\pi^2} = 3\pi\ rad$

Sustituimos las condiciones iniciales para hallar φ_0

$x = A \cdot \operatorname{sen}(\omega t + \varphi_0)$
$x = 0'05\,m, t = 0;\ \omega = 3\pi\ rad;\ A = 0'2\,m$
$\Rightarrow 0'05 = 0'2 \cdot \operatorname{sen}(3\pi \cdot 0 + \varphi_0)$

$3\pi \cdot 0 = 0$

$v =$

$\operatorname{sen}\varphi_0 = 0'25 \rightarrow \varphi_0 = \operatorname{arc\,sen} 0'25 = 0'25\ rad$

$x(t) = 0'2 \cdot \operatorname{sen}(3\pi \cdot t + 0'25)\ m$

$v(t) = 0'2 \cdot 3\pi \cdot \cos(3\pi t + 0'25)\ m/s$

$v(t) = 0'6\pi \cdot \cos(3\pi t + 0'25)\ m/s$

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$m = 0'1\,kg$
$k = 10\ N/m$
$E_M = 1'2\,J$

a) $A = ?$
$T = ?$

$k = m \cdot \omega^2 \rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10\ N/m}{0'1\,kg}} = 10\ rad/s$

$\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = 0'2\pi\ s$

$E_M = \frac{1}{2} m \cdot A^2 \cdot \omega^2 \rightarrow A^2 = \frac{2E_M}{m \cdot \omega^2}$

$A = \sqrt{\frac{2E_M}{m \cdot \omega^2}} = \sqrt{\frac{2 \cdot 1'2\,J}{0'1\,kg \cdot (10\ rad/s)^2}} = 0'49\,m$

b) En $t = 0 \Rightarrow a_{max}$; $x(t) = ?$

$v(5) = ?$

$x = A \cdot \operatorname{sen}(\omega t + \varphi_0)$

$a = -\omega^2 \cdot A \cdot \operatorname{sen}(\omega t + \varphi_0)$

$a_{max} = \omega^2 A$ y puede ocurrir $t = 0 \Rightarrow \operatorname{sen}(\omega t + \varphi_0) = -1$

ya que $a = a_{max} \cdot (-1)$

$\operatorname{sen}\varphi_0 = -1 \Rightarrow \varphi_0 = \frac{3\pi}{2}\ rad = -90°$

$x = 0'49 \cdot \operatorname{sen}\left(10t + \frac{3\pi}{2}\right) m$

$v = \frac{dx}{dt} = 4'9 \cdot \cos\left(10t + \frac{3\pi}{2}\right) m/s$

$v(5) = 4'9 \cdot \cos\left(10 \cdot 5 + \frac{3 \cdot \pi}{2}\right) = -1'29\ m/s$

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$v = 20\,m/s$
$A = 0'25\,m$
$f = 0'2\,Hz$
$\lambda = ?$
$\omega = ?$
$y = ?$
$v_t = ?$
$a = ?$
$K = ?$

$\lambda =$

$v = \frac{\lambda}{T} \Rightarrow v = \lambda \cdot f \Rightarrow \lambda = \frac{v}{f} = \frac{20\,m/s}{0'2\,Hz} = 100\,m$

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{100\,m} = \frac{\pi}{50} = 0'063\,rad/m$

$\omega = 2 \cdot \pi \cdot f = 2 \cdot \pi \cdot 0'2 = 1'26\,rad/s$

$y(x,t) = A \cdot \text{sen}(\omega t + Kx + \phi_0)$

para $t = 2'5\,s$ y $x = 0\,m \Rightarrow y(0, 2'5) = 0'25 \cdot \text{sen}\left(0'4\pi \cdot 2'5 + \frac{\pi}{50} \cdot 0 + \phi_0\right) = 0'25$

$0'25 = 0'25\,\text{sen}(0'4\pi \cdot 2'5 + \phi_0)$

como $\text{sen}(\pi + \phi_0) = 1 \Rightarrow \frac{\pi}{2} = \pi + \phi_0 \Rightarrow \phi_0 = -\frac{\pi}{2}\,rad$

$y(x,t) = 0'25\,\text{sen}\left(1'26t + 0'063x - \frac{\pi}{2}\right)\,m$

$v(x,t) = 0'32\cos\left(1'26t + 0'063x - \frac{\pi}{2}\right)\,m/s$

$a(x,t) = -0'4\,\text{sen}(1'26t + 0'063x - \pi/2)\,m/s^2$

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$P = 1000\,w$
$f = 1000\,MHz$
$r = 200\,km$

$I = \frac{P}{S}$
$P = \frac{E}{t}$
$S = 4\pi r^2$

$I = \frac{P}{4\pi r^2} = \frac{1000\,w}{4\pi (200 \cdot 10^3)^2} = 1'99 \cdot 10^{-9}\,\frac{w}{m^2}$

$I = 1'99 \cdot 10^{-9}\,\frac{w}{m^2}$

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$\beta = 0'1\ cm^{-1}$

$I = \frac{I_0}{10}$

$I = I_0 \cdot e^{-\beta x}$

$\frac{I_0}{10} = I_0 \cdot e^{-\beta x} \Rightarrow \frac{1}{10} = e^{-0'1x} \Rightarrow$

$\Rightarrow \ln\frac{1}{10} = \ln e^{-0'1x} \Rightarrow -\ln 10 = -0'1x \Rightarrow \boxed{x = \frac{\ln 10}{0'1} = 23\ cm}$

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$I_0 = 0'01\ W/m^2$

$x = 1\ m$

$I = \frac{I_0}{2}$

$I = I_0 \cdot e^{-\beta x}$

$\frac{I_0}{2} = I_0 \cdot e^{-\beta x} \Rightarrow \frac{1}{2} = e^{-\beta \cdot x}$

$\ln\left[\frac{1}{2}\right] = \ln e^{-\beta \cdot x} \Rightarrow -\ln 2 = -\beta x \Rightarrow$

$\Rightarrow \boxed{\beta = \frac{\ln 2}{1m} = 0'69\ m^{-1}}$

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a) $f = \frac{1}{T} = \frac{1}{2} = 0'5\ Hz$

$\lambda = \frac{v}{f} = \frac{0'6\ m/s}{0'5\ Hz} = 1'2\ m$

$y(x,t) = A\ sen\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \varepsilon_0\right]$

b) $y(0'3,1) = 0 = A \cdot sen\left[2\pi\left(\frac{1}{2} - \frac{0'3}{1'2}\right) + \varepsilon_0\right] \Rightarrow sen\left(\pi - \frac{\pi}{2} + \varepsilon_0\right) = 0$

$\frac{\pi}{2} + \varepsilon_0 = \pm n\pi \Rightarrow \varepsilon_0 = -\frac{\pi}{2} \pm n\pi\ ,\ \forall n = 0,1,2\ldots$

$v(x,t) = A \cdot \frac{2\pi}{T} \cos\left[2\pi\left(\frac{t}{\lambda} - \frac{x}{\lambda}\right) + \varepsilon_0\right] \Rightarrow v(0'3,1) > 0 \Rightarrow$

$\Rightarrow \cos\left[2\pi\left(\frac{1}{2} - \frac{0'3}{1'2}\right) + \varepsilon_0\right] > 0$

$y(0'3,1) = 0'05 = A \cdot sen\left[2\pi\left(\frac{1'5}{2} - \frac{0'3}{1'2}\right) - \frac{\pi}{2}\right] =$

$\boxed{A = 0'05\ m}$

$\cos\left(\frac{\pi}{2} + \varepsilon_0\right) > 0 \Rightarrow -\frac{\pi}{2} < \frac{\pi}{2} + \varepsilon_0 < \frac{\pi}{2}$

$\Rightarrow -\pi < \varepsilon_0 < 0 \Rightarrow \boxed{\varepsilon_0 = -\frac{\pi}{2}}$

ya que antes salió $\varepsilon_0 = -\frac{\pi}{2} \pm n\pi$

c) $\Delta\varepsilon = \left(\omega t - \frac{2\pi}{\lambda} x_1 + \varepsilon_0\right) - \left(\omega t - \frac{2\pi}{\lambda} x_2 + \varepsilon_0\right) =$

$= \frac{2\pi}{\lambda} \Delta x$

si $\Delta x = \frac{\lambda}{4} \Rightarrow \Delta\varepsilon = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{2 \cdot \pi}{4} = \frac{\pi}{2}\ rad.$